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ONSET OF SLUGGING IN HORIZONTAL DUCTS

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Abstract—The prediction afforded by Gardner's (1977) hypothesis for the onset of slugging is shown to form an upper bound to the data of Kordyban (1977) for both the required phase velocity difference and the wave height for conditions just subcritical to those needed for slugging. Experimental data is only available for air/water flow at atmospheric pressure in ducts of rectangular cross-section. Predictions are presented for fluids with other density ratios in similar ducts and in horizontal tubes.

1. INTRODUCTION

Slugging in horizontal ducted two-phase flow systems is a phenomenon which sets in suddenly as, say, the light phase flowrate is slowly increased and the suddeness is a factor which should be explained in any theory. The problem has been studied theoretically and experimentally by Kordyban & Ranov (1970) and Wallis & Dobson (1973). Theoretical treatments have been given by Taitel & Dukler (1976) and Gardner (1977). Kordyban (1977) has recently produced fresh experimental data including, for the first time, information on the height of waves present for conditions which are just sub-critical to those causing slugging. This last data allows a more complete comparison of the various theories.

The experimental information, although limited to air/water systems at atmospheric pressure, is excellent and there is substantial agreement between different workers. However, the theoretical treatments, due to Wallis & Dobson (1973) and Taitel & Dukler (1976), are open to criticism, as, at least, the former workers admit. It is helpful to refer to figure 1 which will be used in the theoretical work of this paper. A basic flaw in both the above theories is that it is assumed that the water velocity is the same at station 1 before a wave and at station 2 in a wave. Since the basic water flow, which is that at station 1, was held zero in experiments, it is therefore assumed that the waves also have a zero velocity. Experimentally this was manifestly untrue.

Wallis and Dobson continued by saying that the water in the wave above the level at station 1 in figure 1 could be regarded as the inverse of Benjamin's (1968) long bubble held stationary by a water flow. This is true. The water in the wave can be regarded as equivalent to the bubble and it can be held stationary by the air flow. Using Benjamin's analytical result,

$$u_{1L}\left[\frac{\rho_L}{\Delta\rho g(H-h_1')}\right]^{1/2}=0.5 \qquad [1.1]$$

which, it was asserted, gives the critical conditions for the onset of slugging. Subscript 1 indicates station 1, subscripts L and H indicate light and heavy phases, u is velocity, ρ is density, $\Delta\rho$ is the density difference between the phases, H is the total channel depth, h' is the depth of the heavy phase and g is acceleration due to gravity.

A corollary from Benjamin's work, which Wallis and Dobson mention, is that the height of the wave $(h_2'-h_1')$ just before slugging sets in should be half the depth of the light phase at station 1 or

$$h_2'-h_1'=\frac{1}{2}(H-h_1').$$

This will later be compared with Kordyban's (1977) recent data.

It is seen that there is nothing in Wallis and Dobson's explanation which indicates the sudden onset of slugging, when the wave suddenly bridges the whole channel height. If one follows their line of thought, a slight increase or decrease of the light phase velocity will make the wave move one way or the other to maintain the light phase velocity relative to the wave approximately constant. It will be kept exactly constant if the water velocity over the depth h_1' remains unchanged, even beneath the waves, as Wallis and Dobson asserted in order to extend the range of [1.1] by writing

$$(u_{1L} - u_{1H})\left[\frac{\rho_L}{\Delta\rho g(H - h_1')}\right]^{1/2} = 0.5. \qquad [1.3]$$

Taitel & Dukler (1976) started by examining the condition for holding a wave with an infinitesimally small height stationary. Gardner (1977) gives the general expression, which for a rectangular channel becomes

$$\frac{\rho_H u_H^2}{\Delta\rho g h'} + \frac{\rho_L u_L^2}{\Delta\rho g(H - h')} = 1 \qquad [1.4]$$

Equation [1.4], for the heavy phase velocity u_H set equal to zero, becomes [1.1], with the left hand side set equal to unity instead of a half. Taitel and Dukler continued by showing that larger waves, which were also stationary, were less stable and thus [1.4] with $u_H = 0$ provides the criterion for the onset of slugging. This, of course, begs the question as to why larger waves should not move, especially when they are observed to move experimentally. However, the biggest difficulty is that the criterion predicts velocities that are twice those predicted by Wallis and Dobson, whose equation is in fair agreement with the results. Therefore Taitel and Dukler speculated that they should not consider infinitesimally small waves but a wave whose height increases with the depth of the light phase. They arbitrarily stated that the criterion was therefore

$$u_L\left[\frac{\rho_L}{\Delta\rho g(H - h')}\right]^{1/2} = H - h'. \qquad [1.5]$$

The reasoning employed by Taitel and Dukler also leads one to expect that the wave height is given by

$$\frac{h_2' - h_1'}{H - h_1'} = 1 - 0.25\left[1 - \frac{h_1'}{H}\right]^2 - 0.25\left[1 - \frac{h_1'}{H}\right]\left[\left[1 - \frac{h_1'}{H}\right]^2 + 8\right]^{1/2} \qquad [1.6]$$

and this equation will be checked against Kordyban's (1977) data later.

Lastly it is noted with respect to Taitel & Dukler's (1976) theory that there is again nothing in it leading one to expect a sudden onset of slugging, since waves have an ability to move and setting their velocity equal to zero is quite arbitrary.

Gardner (1977) sketched in his theory briefly as an example of the use of a theory of lossless waves. The present paper will make a complete presentation and compare the results and those of the theories discussed above with Kordyban's (1977) results. The theory has already been compared favourably with the results of Kordyban & Ranov (1970) and Wallis & Dobson (1973), (Gardner 1977).

The lossless wave system was derived from the equations of conservation of energy for both phases and an equation for the conservation of momentum for the whole flow. It was assumed that velocity is uniform with depth in each phase at stations 1 and 2, which are well removed upstream and downstream from the wave, as shown in figure 1, and that there is no

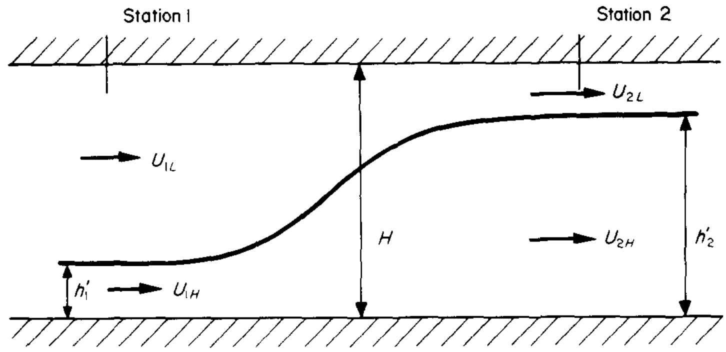

Figure 1. Definition sketch of system with the wave stationary.

friction. This type of wave is assumed to exist in the two-phase flow in question. As a result, waves move with a prescribed velocity with respect to the stationary heavy phase.

An additional hypothesis is required to derive the criterion for the onset of slugging. It is imagined that the light phase flow is driving the waves and, therefore, co-ordinates are chosen such that the heavy phase is stationary at station 1 in figure 1 and the energy flux through station 1, due.solely to the light phase, is calculated. Similarly the energy flux through station 2 within the wave is calculated. This flux is due to both phases, since the wave is moving.

Now, if we plot the energy flux through station 2 less that through station 1 vs the difference in velocities between the phases at station 1 while keeping the heavy phase depth constant at station 1, we obtain graphs like those in figure 2, which is taken from Gardner (1977)

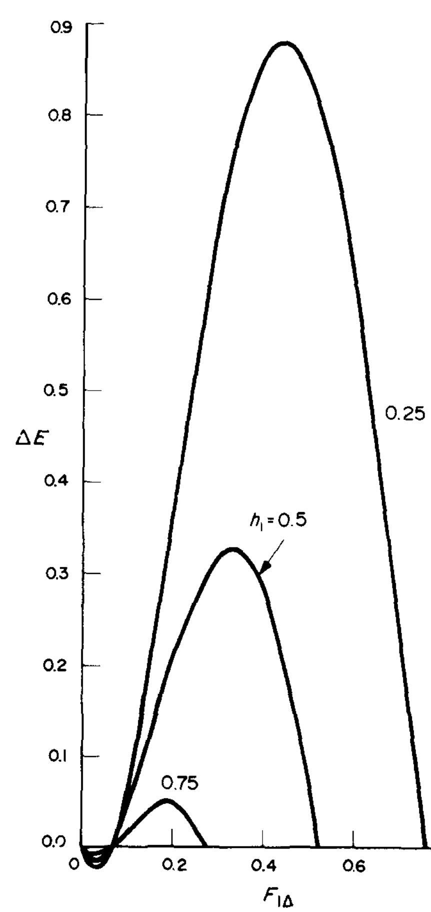

Figure 2. Energy flux for air/water system.

(N.B. $F_{1\Delta}$ is proportional to the velocity difference). The hypothesis is that the system described is valid as long as the energy difference increases with the velocity difference, it being considered unreasonable that an increase in the velocity difference will reduce the energy put into the wave system and eventually lost. The maxima of the curves in figure 2 therefore define a change of flow regime and it is assumed that they define the onset of slugging. It is noticed that not only is the improbable restriction of setting the wave velocity to zero removed but, also, the choice of a maximum to define the criterion explains the suddenness of the onset.

It remains to note that the energy difference is seen to be negative near the origin of figure 2 and, consequently, maxima may appear with a negative energy difference. The region is quite small for air/water systems at atmospheric pressure and has not been experimentally examined. However, it seems improbable that slugging will occur there and, perhaps, the system is then quiescent.

2. THEORY FOR CLOSED CHANNELS OF RECTANGULAR CROSS-SECTION

2.1 *Onset of slugging*

The system is illustrated in figure 1, where co-ordinates are chosen such as to maintain the wave stationary. Subscripts L and H refer to the light and heavy phases respectively and subscripts 1 and 2 represent stations at which the heavy phase is shallow and deep respectively. u is velocity, h' is depth of the heavy phase, ρ is density and $\Delta\rho$ is the density difference between the fluids.

The wave illustrated in figure 1 is lossless and therefore the energy flux for each fluid at station 1 equals that at station 2. The energy per unit volume of the light fluid referred to the top of the channel is

$$\frac{\rho_L u_{1L}^2}{2} + \Delta p - \rho_L g(H - h_1') = \frac{\rho_L u_{2L}^2}{2} - \rho_L g(H - h_2') \qquad [2.1]$$

where H is the channel depth, Δp is the pressure at the interface at station 1 and the pressure is arbitrarily zero at the interface at station 2.

The co-ordinate system is now changed to one in which the heavy phase is stationary at station 1. The energy fluxes at the two stations then become different to each other. The flux at station 1, referred to the level of the interface at station 2, is

$$\left[\rho_L \frac{(u_{1L} - u_{1H})^2}{2} + \Delta p - \rho_L g(h_2' - h_1')\right](u_{1L} - u_{1H})(H - h_1'). \qquad [2.2]$$

The energy flux at station 2 is

$$\rho_L \frac{(u_{2L} - u_{1H})^3}{2}(H - h_2') + \rho_H \frac{(u_{2H} - u_{1H})^3}{2} h_2'. \qquad [2.3]$$

Equation [2.1] is used to eliminate Δp from [2.2] and the result is subtracted from [2.3] to obtain the required energy difference Δe. The result next requires substitutions from equations defining the properties of the lossless wave system. First, we make the definitions

$$F_{1\Delta} = \left(\frac{\rho_L}{\Delta\rho g H}\right)^{1/2}(u_{1L} - u_{1H}) \qquad [2.4]$$

$$h = \frac{h'}{H} \qquad [2.5]$$

$$P = (\rho_H/\rho_L)^{1/2}. \qquad [2.6]$$

Then the properties of the wave system are expressed by (Gardner 1977)

$$h_2 = -\left(\frac{\rho_H}{\Delta\rho g H}\right)^{1/2} u_{1H} \qquad [2.7]$$

$$h_1 = -\left(\frac{\rho_H}{\Delta\rho g H}\right)^{1/2} u_{2H} \qquad [2.8]$$

$$(1 - h_2) = \left(\frac{\rho_L}{\Delta\rho g H}\right)^{1/2} u_{1L} \qquad [2.9]$$

$$(1 - h_1) = \left(\frac{\rho_L}{\Delta\rho g H}\right)^{1/2} u_{2L} \qquad [2.10]$$

all of which essentially state that a particular form of Froude number at one station equals plus or minus the reduced depth of that phase at the other station. Plus or minus signs can be chosen to suit the problem.

Another equation which was derived by Gardner (1977) from [2.7] to [2.10] is

$$h_2 = \frac{1 - F_{1\Delta}}{\left(1 - \frac{1}{P}\right)}. \qquad [2.11]$$

There is now sufficient information to transform the expression for the energy flux difference Δe, after considerable algebraic manipulation, into

$$\frac{P^2}{(P-1)^3}\Delta E = h_2(h_2 - h_1)\left[(1 - h_2)^2 - \left(\frac{h_2}{P}\right)^2\right] \qquad [2.12]$$

where

$$\Delta E = \frac{2(P-1)^3}{P}\frac{\rho_L{}^{1/2}\Delta e}{H(\Delta\rho g H)^{3/2}} \qquad [2.13]$$

h_1 and h_2 have been chosen as the independent variables in [2.12] but this equation is readily transformed into one with h_1 and $F_{1\Delta}$ as the independent variables and the result is

$$\Delta E = F_{1\Delta}(1 - F_{1\Delta})\left[1 - F_{1\Delta} - \left(1 - \frac{1}{P}\right)h_1\right][(1+P)F_{1\Delta} - 2] \qquad [2.14]$$

which is the equation given by Gardner (1977), though with a typographical error corrected.

Differentiation of [2.14] with respect to $F_{1\Delta}$ with h_1 held constant and then setting the differential equal to zero gives the condition for the onset of slugging

$$1 - \left(1 - \frac{1}{P}\right)h_1 = \frac{4F_{1\Delta} - 3(3+P)F_{1\Delta}^2 + 4(1+P)F_{1\Delta}^3}{2 - 2(3+P)F_{1\Delta} + 3(1+P)F_{1\Delta}^2}. \qquad [2.15]$$

Now examination of [2.14] shows that ΔE is negative when

$$(1+P)F_{1\Delta} > 2. \qquad [2.16]$$

Elimination of P between [2.15] and inequality [2.16] gives

$$F_{1\Delta} > 2(1 - h_1) \qquad [2.17]$$

as defining the region where negative values of the maximum of $F_{1\Delta}$ are found. As stated in the Introduction, slugging will probably not occur in this region.

Figure 3 shows the prediction for the onset of slugging in co-ordinates of the velocity difference Froude number, $F_{1\Delta}$, vs the reduced depth of the light phase, $(1-h_1)$, with the density ratio number, P, as a parameter. It is noticed that the value of the critical Froude number increases as the density of the light phase increases with respect to that of the heavy phase. Simultaneously, the range of reduced light phase depths, over which slugging is probable also reduces. It may also be noted that it becomes more difficult in practice to attain large velocity differences as the light phase becomes denser and, for this reason as well, slugging may not then be so frequently observed.

2.2 *Wave height at the onset of slugging*

Equation [2.12] is differentiated with respect to h_2 with h_1 held constant to obtain the criterion for the onset of slugging in the form

$$(h_2-h_1)=\frac{h_2(1-h_2)^2-\dfrac{h_2^3}{P^2}}{(3h_2-1)(1-h_2)+3\left(\dfrac{h_2}{P}\right)^2}. \qquad [2.18]$$

Also

$$(1-h_1)=(1-h_2)+(h_2-h_1). \qquad [2.19]$$

Simultaneous solution of [2.18] and [2.19] give the predicted wave height at the onset of slugging.

3. COMPARISON OF THEORY WITH EXPERIMENTAL DATA

The experimental data of Kordyban (1977) for the onset of slugging are plotted in figure 4 in terms of $F_{1\Delta}$ vs $(1-h_1)$, using linear scales. This seems the fairest representation since the

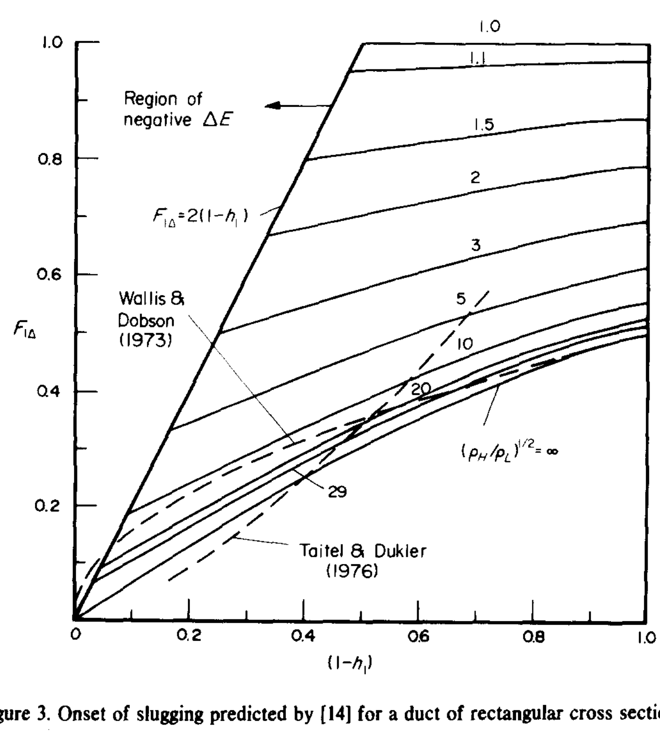

Figure 3. Onset of slugging predicted by [14] for a duct of rectangular cross section.

only experimental variable in $F_{1\Delta}$ is the velocity difference between the phases and $(1-h_1)$ is the reduced light phase depth. Previously, the co-ordinates first chosen by Wallis & Dobson (1973) of $F_{1\Delta}(1-h_1)$ vs $(1-h_1)$, using logerithmic scales, have usually been employed but the danger of seeing a spurious correlation with $(1-h_1)$ in the ordinate is obvious.

Figure 4 shows that the present theory provides a satisfactory upper bound to the experimental points. This is not unreasonable considering that the experimental technique was to slowly raise the air flowrate, with all other parameters kept constant, until slugging set in. It is also reasonable that forcing waves on the system with a wavemaker allowed slugging to set in at lower air flowrates than otherwise. Wallis & Dobson's (1973) theory is seen to predict the onset of slugging to occur at higher flowrates than observed experimentally. Taitel & Dukler's (1976) theory does not follow the trend of the experimental results and becomes seriously in error for large reduced light phase depths.

Figure 5 shows the experimental data of Kordyban (1977) on wave height for conditions just sub-critical for slugging. The reduced wave height (h_2-h_1) is plotted vs the reduced light phase depth $(1-h_1)$.

The theory of this paper shown in figure 5 appears to form a satisfactory upper bound to the experimental points, just as the theory formed an upper bound in figure 4. However, the theory of Wallis & Dobson (1973) predicts substantially higher wave heights and Taitel & Dukler's (1976) theory predicts wave heights even higher again. Indeed, Taitel & Dukler seem to predict heights for which the wave will almost fill the channel when $(h_2-h_1)=(1-h_1)$. However, it may be noted that Taitel and Dukler's prediction passes through a maximum of $(h_2-h_1)=0.291$ at $(1-h_1)=0.536$ and that (h_2-h_1) falls to zero at $(1-h_1)=1$.

4. THEORY FOR A HORIZONTAL TUBE

The technical application of the theories is to a tube rather than a duct with a rectangular cross-section. First consider a tube of arbitrary cross-sectional shape. Reduce areas by dividing

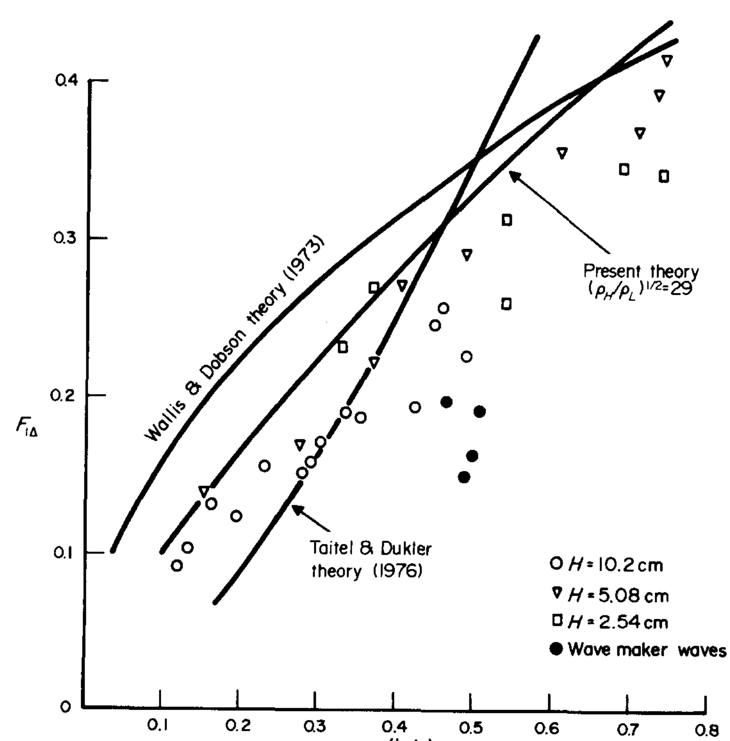

Figure 4. Reduced phase velocity difference versus reduced air depth for onset of slugging. Experimental data of Kordyban (1977).

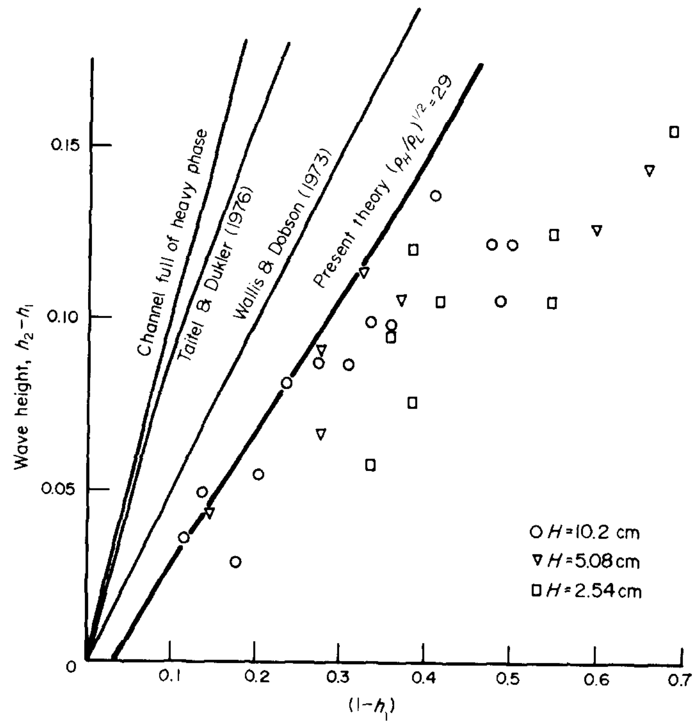

Figure 5. Theory for wave height at onset of slugging compared with data of Kordyban (1977).

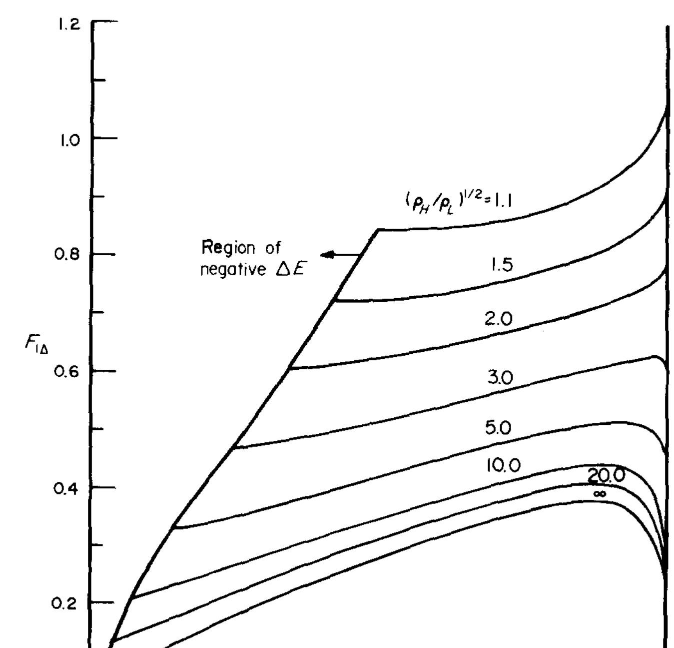

Figure 6. Predicted conditions for the onset of slugging in a horizontal tube of circular cross-section.

them by the total cross-sectional area and reduce lengths by dividing them by any suitable fixed length scale, which is the diameter, d, when the cross-section is a circle. It is then found, by using Gardner (1977), that we need to maximize the function

$$\frac{BA_2}{(A_2 - A_1)^2}\left[B^2\left[(1-A_2)^2 - \left(\frac{A_2}{P}\right)^2\right] - (1-A_1)(1-2A_2+A_1)(h_2-h_1-2y_0)\right] \qquad [4.1]$$

where

$$B = [(1-A_1)(h_2-h_1) - (2-A_1-A_2)\,y_0]^{1/2}. \qquad [4.2]$$

Here A_1 and A_2 are the reduced areas of the heavy phase at stations 1 and 2 and y_0 is the reduced distance of the centre of pressure of $(A_2 - A_1)$ from the interface at station 2.

Results for a tube of circular cross-section are given in figure 6 where

$$F_{1\Delta} = \left(\frac{\rho_L}{\Delta\rho g d}\right)^{1/2}(u_{1L} - u_{1H}). \qquad [4.3]$$

5. CONCLUDING REMARK

The theory for the onset of slugging presented by this paper appears satisfactory compared with results for air/water systems at atmospheric pressure. However, experimental results for which the ratio of the densities of the two phases are varied substantially are required before it can be concluded that the theory is entirely satisfactory.

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